analogue electronics

protocol for the laboratory work

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31st January 2004

Contents

1. Operational Amplifiers

The term operational amplifier or "op-amp" refers to a class of high-gain DC coupled amplifiers with two inputs and a single output. Some of the general characteristics of the IC version are: [7]

- High gain, on the order of a million
- High input impedance, low output impedance
- Used with split supply, usually ± 15V
- Used with feedback, with gain determined by the feedback network.
- zero point stability
- defined frequency response

Their characteristics often approach that of the ideal op-amp and can be understood with the help of the golden rules.

The Ideal Op-amp

The IC Op-amp comes so close to ideal performance that it is useful to state the characteristics of an ideal amplifier without regard to what is inside the package. [7]

- Infinite voltage gain
- Infinite input impedance ($r_e = \mathrm{d}U_e/\,\mathrm{d}I_e \to \infty$)
- Zero output impedance ($r_a = \mathrm{d}U_a/\,\mathrm{d}I_a \to 0$)
- Infinite bandwidth
- Zero input offset voltage (i.e., exactly zero out if zero in).

These characteristics lead to the golden rules for op-amps. They allow you to logically deduce the operation of any op-amp circuit.

The Op-amp Golden Rules

From Horowitz & Hill: For an op-amp with external feedback

I. The output attempts to do whatever is necessary to make the voltage difference between the inputs zero.

II. The inputs draw no current.

properties of the Op-amp

Figure 1 shows the circuit-symbol of an Operational Amplifier. The Input of an Op-amp is a differential amplifier, which amplifies the difference between both inputs.

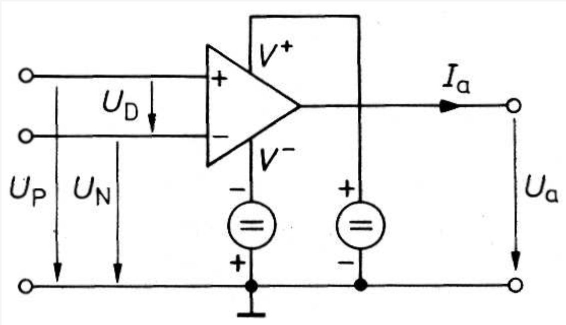


Figure 1: circuit-symbol of the OP-amp [8]

If on both inputs the same voltage is applied the output will be zero in the ideal case. Whereas a difference leads to an output signal of

$$U_a = G(U_P - U_N)$$

with the differential-gain G. For this reason the P-input is called the non-inverting input and labelled with a plus-sign and contrary the N-input called the inverting input labelled with a minus-sign.

LM 741 OP

We are using the LM741 operational amplifier. The chip has 8 pins used to both power and use the amplifier. The pinout for the LM741 are listed below:

pin	name	description
1	NULL	Offset Null
2	V_-	Inverting Input
3	V_+	Non-Inverting Input
4	$-V_{CC}$	Power (Low)
5	NULL	Offset Null
6	V_{Out}	Output Voltage
7	$+V_{CC}$	Power (High)
8	NC	Not Connected

Table 1: pinout for the LM741

2. Circuits with Operational Amplifiers

2.1. Inverting Operational Amplifier

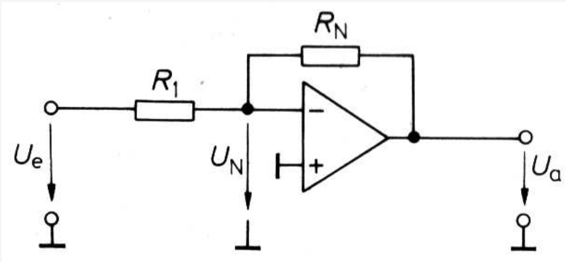


Figure 2: inverting amplifier circuit [8]

Calculation of output voltage

1. high input impedance $r_e \to \infty$: $I_1 = I_N = I$
2. the feedback attempts to make the voltage difference between the inputs zero: $U_P - U_N = 0V$. We use therefore Kirchhoff's node law to calculate the output voltage that is necessary to lead U_N to zero.

$$\left.\begin{aligned} U_a &= -IR_N \\ U_e &= \quad IR_1 \end{aligned}\right\} \Rightarrow \frac{U_e}{R_1} = -\frac{U_a}{R_N} = 0 \Rightarrow U_a = -\frac{R_N}{R_1}U_e$$

The gain is thus $g = -\frac{R_N}{R_1}$ with a phaseshift of 180°.

This means that the OP acts in such a way that the output voltage U_a is adjusted so that the negative input is set to $U_N = 0$. The N-input acts thus like a ground.

If we assume the resistances to be general impedances the gain still remains the same, just with different values: $g = -Z_2/Z_1$. This still holds even if the impedances mean a complex circuits itsself.

We setup an operational amplifier with proportional gain of 10.
The electronic devices used are:

1. $Z_1 = 10.11\,\mathrm{k}\Omega$
2. $Z_2 = 100.2\,\mathrm{k}\Omega$

$$g = -\frac{Z_2}{Z_1} = -\frac{100.2}{10.11} \approx -10$$

The minus sign in the gain denotes the phase shift of 180° between input and output voltage.

We want to record the amplification and phase shift spectrum. Therefore we scan the amplification over the frequency range and take values at approximately equal distances on a logarithmic scale. The phase shift is calculated using: $\Delta\varphi = 2\pi\nu \cdot \Delta T$ with time difference ΔT between the to signals on the oscilloscope. The plots are presented in figures 3 and 4.

The gain of an real amplifiers is not constant over the whole frequency spectrum as can be seen in the figure 3. It starts to decrease rapidly at about 10 kHz. The measured phase shift is shown in figure 4. The data has been checked, but we have really measured these values. These values however do not represent the shape that would be expected. That would be a change by 180° from 180° to 360° over the whole range.

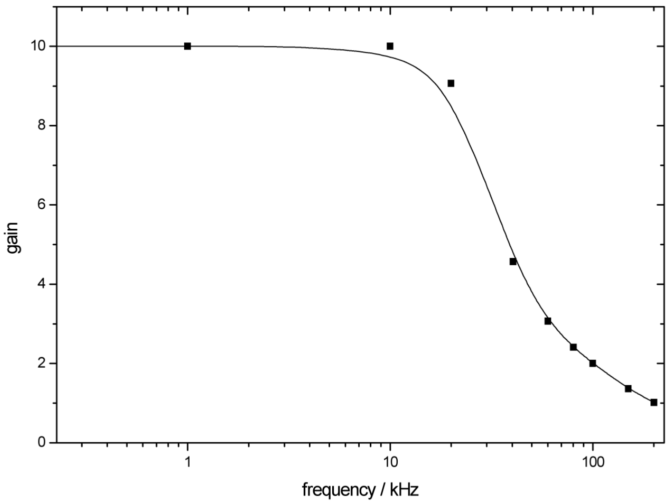


Figure 3: gain spectrum of Inverting Amplifier

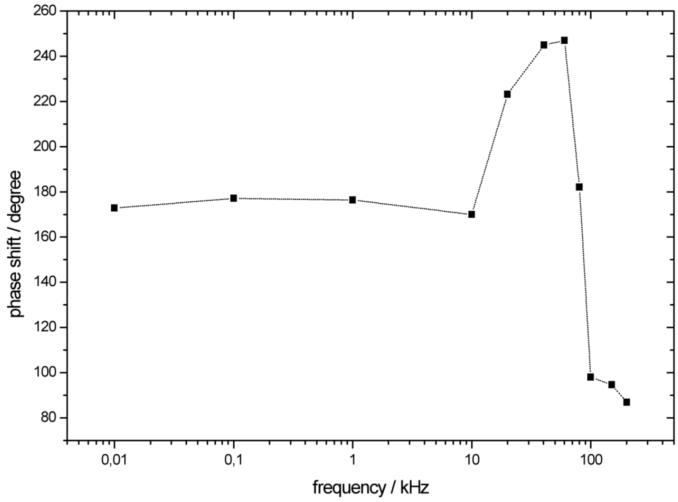


Figure 4: phase change spectrum of Inverting Amplifier

2.2. Integrator

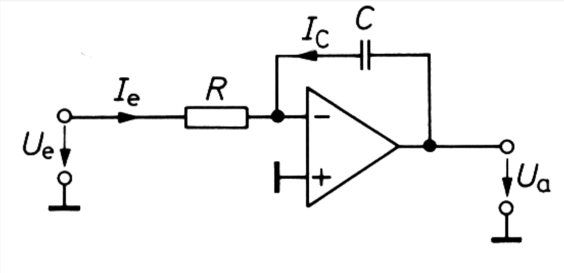


Figure 5: integrator circuit [8]

time dependence of output:

1. current depends on voltage: $Q = CU_a \Rightarrow I = \dot{Q} = C\dot{U}_a$
2. current in circuit is constant: $I = \frac{U_e}{R} = -C\dot{U}_a$

$$U_a = \frac{1}{RC}\int U_e \, \mathrm{d}t + U_a(t_0)$$

gain:

$$g = -\frac{Z_N}{Z_1} = -\frac{\frac{1}{i\omega C}}{R} = -\frac{1}{i\omega RC}$$

phase:

$$Z = Z_1 + Z_2 = R + \frac{1}{i\omega C} = R + i\left(-\frac{1}{\omega C}\right)$$

$$\tan\varphi = \frac{\mathrm{Im(Z)}}{\mathrm{Re(Z)}} = -\frac{1}{\omega RC}$$

$$\Rightarrow \varphi = \arctan\left(-\frac{1}{\omega RC}\right)$$

We setup an Integrator circuit with a cut-off frequency at about 500Hz.
The electronic devices used are:

1. $R_1 = 4.69\,\mathrm{k\Omega}$
2. $R_2 = 10.06\,\mathrm{k\Omega}$
3. $Z_1 = R_1 \parallel R_2 = 3200\,\Omega$
4. $Z_2 = 100\,\mathrm{nF}$

The cut-off frequency is defined as $g(\nu) = 1$.

$$g = -\frac{Z_2}{Z_1} = -\frac{1}{i\omega RC} \Rightarrow |g| = \frac{1}{2\pi\nu RC}$$

With this setup we achieve thus a frequency of

$$\nu = \frac{1}{2\pi RC} \approx 497\text{Hz}$$

The phase follows the function

$$\varphi = \arctan\left(-\frac{1}{\omega RC}\right) = \arctan\left(-\frac{1}{\nu} \cdot 497\,\text{Hz}\right)$$

Figures 6 and 7 show the plots for gain and phase. The gain follows obviously very perfectly the theoretical curve. Whereas the phase is completely useless. The reason for this behaviour is unknown. Since we do not get usefull values for at least one of the circuits we must assume that we have done a systematical error in the measurement, although it is unclear to us what should have been done differently.

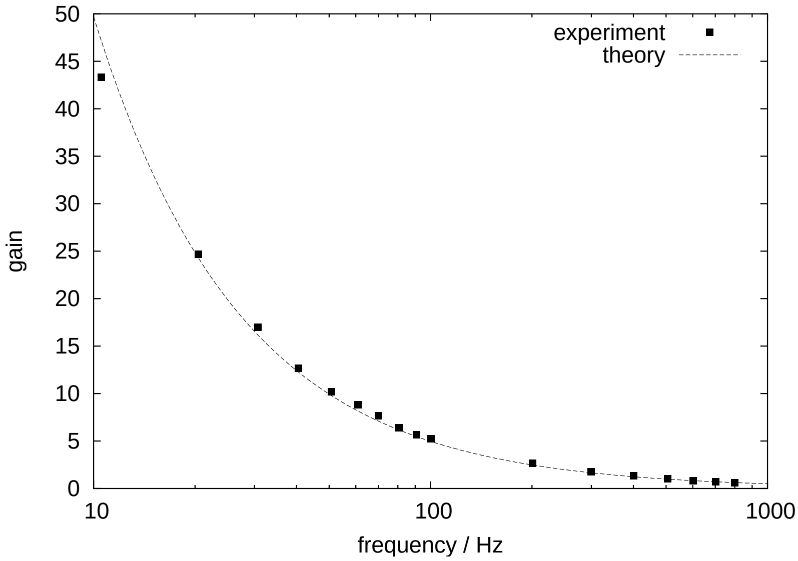


Figure 6: gain spectrum of Integrator

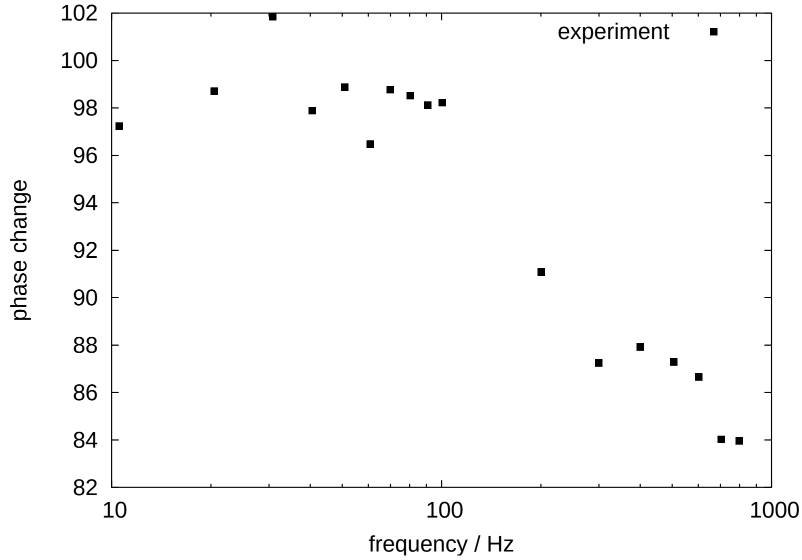


Figure 7: phase change spectrum of Integrator

2.3. Differentiator

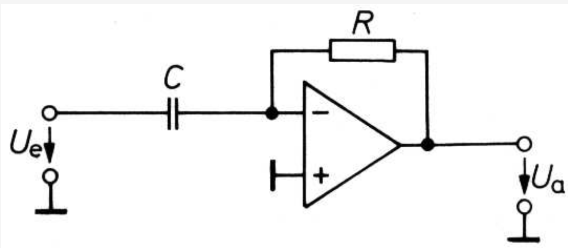


Figure 8: differentiator circuit [8]

time dependence of output:

1. current depends on voltage: $Q = CU_e \Rightarrow I = \dot{Q} = C\dot{U}_e$
2. current in circuit is constant: $I = -\frac{U_a}{R} = C\dot{U}_e$

$$U_a = -RC \cdot \dot{U}_e$$

gain:

$$g = -\frac{Z_N}{Z_1} = -\frac{R}{\frac{1}{i\omega C}} = -i\omega RC$$

phase:

$$
\begin{aligned}
Z &= Z_1 + Z_2 = R + \frac{1}{i\omega C} = R + i\left(-\frac{1}{\omega C}\right) \\
\tan\varphi &= \frac{\mathrm{Im(Z)}}{\mathrm{Re(Z)}} = -\frac{1}{\omega RC} \\
\Rightarrow \varphi &= \arctan\Big(-\frac{1}{\omega RC}\Big)
\end{aligned}
$$

We setup an Differentiator circuit with a cut-off frequency at about 5000 Hz.
The electronic devices used are:

1. $R_1 = 4.69\,\mathrm{k\Omega}$
2. $R_2 = 10.07\,\mathrm{k\Omega}$
3. $Z_2 = R_1 \parallel R_2 = 3200\,\Omega$
4. $Z_1 = 10\,\mathrm{nF}$

The cut-off frequency is defined as $g(\nu) = 1$.

$$
g = -\frac{Z_2}{Z_1} = -i\omega RC \;\Rightarrow |g| = 2\pi\nu RC
$$

With this setup we achieve thus a frequency of

$$
\nu = \frac{1}{2\pi RC} \approx 4970\mathrm{Hz}
$$

The phase follows the function

$$
\varphi = \arctan\Big(-\frac{1}{\omega RC}\Big) = \arctan\Big(-\frac{1}{\nu}\cdot 4970\,\mathrm{Hz}\Big)
$$

Figures 9 and 10 show the plots for gain and phase. The plot of gain proves the increase in gain with frequency as predicted by the theory. The decrease starting at about 20 kHz is due to the inherent properties of the operational amplifier. This decrease has the same origin as the one that we observed in section 2.1 on page 5. The phase however does not coincidence with the theory as already discussed in section 2.2 on page 8.

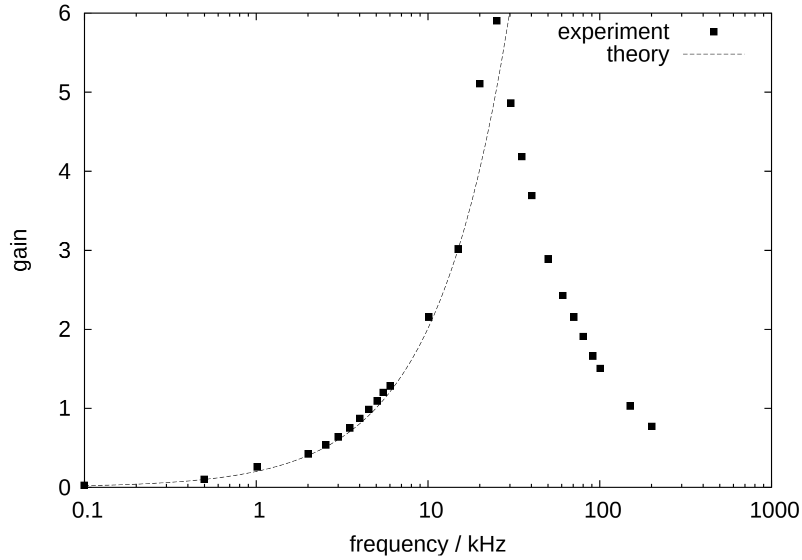


Figure 9: gain spectrum of Differentiator

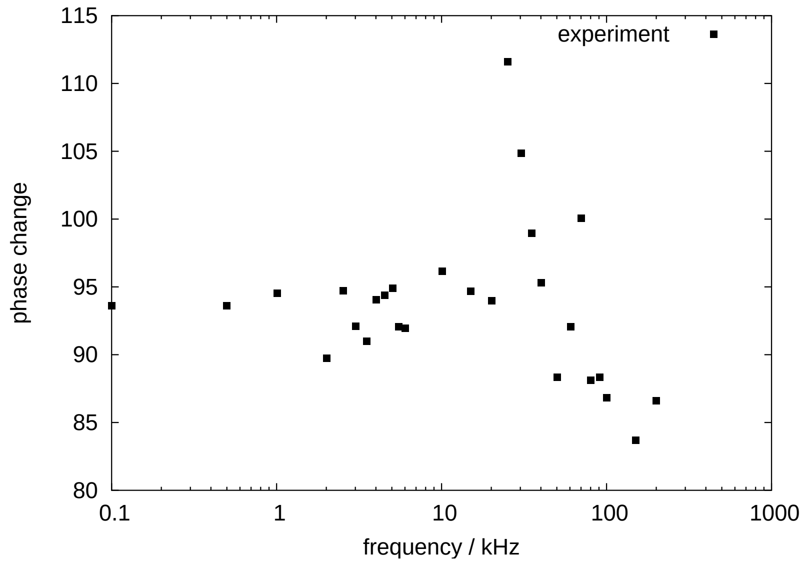


Figure 10: phase change spectrum of Differentiator

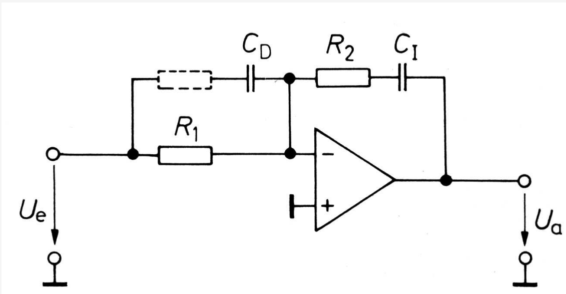


Figure 11: PID controller circuit [8]

Calculation of gain

$$
\begin{aligned}
Z_1 &= \left(\frac{1}{R_1} + \frac{1}{\frac{1}{i\omega C_D}}\right)^{-1} = \frac{R_1 \cdot \frac{1}{i\omega C_D}}{R_1 + \frac{1}{i\omega C_D}} = \frac{R_1}{i\omega R_1 C_D + 1} \\
Z_2 &= R_2 + \frac{1}{i\omega C_I}
\end{aligned}
$$

inserted in the gain definition

$$
\begin{aligned}
g &= -\frac{Z_2}{Z_1} = -\frac{1}{R_1}\left(R_2 + \frac{1}{i\omega C_I}\right)(i\omega R_1 C_D + 1) \\
&= -\left[\frac{R_2}{R_1} + \frac{C_D}{C_I} + i\left(\omega R_2 C_D + \frac{1}{\omega R_1 C_I}\right)\right]
\end{aligned}
$$

assuming $\frac{C_D}{C_I} \ll \frac{R_2}{R_1}$

$$
g = -\frac{R_2}{R_1}\left[1 + i\left(\omega R_1 C_D + \frac{1}{\omega R_2 C_I}\right)\right]
$$

Calculation of phase:

$$
\begin{aligned}
Z = Z_1 + Z_2 &= \frac{R_1}{i\omega R_1 C_D + 1} + R_2 + \frac{1}{i\omega C_I} \\
&= \frac{R_1}{1 + (\omega R_1 C_D)^2} + R_2 + i\left\{-\frac{\omega C_D R_1^2}{1 + (\omega R_1 C_D)^2} - \frac{1}{\omega C_I}\right\} \\
\tan\varphi &= \frac{\mathrm{Im(Z)}}{\mathrm{Re(Z)}} = \ldots = \frac{\omega^2 R_1^2 C_D (C_I - 1) - 1}{R_1 + R_2 + \omega^2 R_1^2 R_2 C_D^2}
\end{aligned}
$$

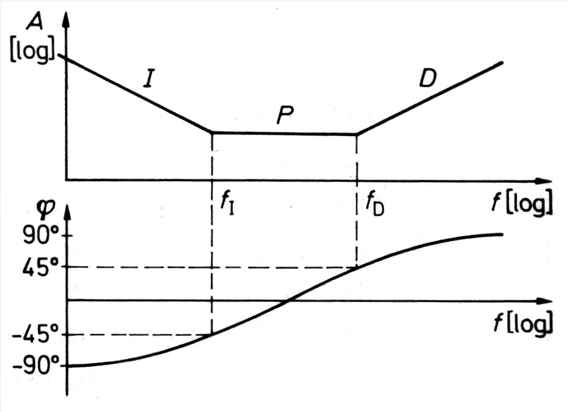


Figure 12: Bode diagram of a PID controller [8]

We combine the three servos to a PID servo.
The electronic devices used are:

1. $R_1 = 3.18\,\mathrm{k}\Omega = 10\,\mathrm{k}\Omega \parallel 4.7\,\mathrm{k}\Omega$
2. $R_2 = 3.18\,\mathrm{k}\Omega = 10\,\mathrm{k}\Omega \parallel 4.7\,\mathrm{k}\Omega$
3. $C_I = 100\,\mathrm{nF}$
4. $C_D = 10\,\mathrm{nF}$

Here we have now two cut-off frequencies at $\nu_I = 497\mathrm{Hz}$ and $\nu_D = 4970\mathrm{Hz}$.
The gain follows

$$
\begin{aligned}
g &= -\left[\frac{R_2}{R_1} + \frac{C_D}{C_I} + i\left(\omega R_2 C_D + \frac{1}{\omega R_1 C_I}\right)\right] \\
|g(\nu)| &= \left\{(1.1)^2 + \left(\frac{1}{4970\,\mathrm{Hz}}\cdot\nu + \frac{497\,\mathrm{Hz}}{\nu}\right)^2\right\}^{-1/2}
\end{aligned}
$$

The phase follows the function

$$
\varphi = \arctan\left(\frac{\omega^2 R_1^2 C_D (C_I - 1) - 1}{R_1 + R_2 + \omega^2 R_1^2 R_2 C_D^2}\right) = \arctan\left(\frac{-\omega^2\cdot 1.011 - 1}{6360 + 1.011\cdot 10^8\cdot\omega^2}\right)
$$

Figures 13 and 14 show the plots for gain and phase. The plot of gain proves the combined behaviour of all three circuits; The decrease of the Integrator, the constant gain of the inverting amplifier and the increase of the Differentiator. The decrease starting at about 20 kHz is due to the inherent properties of the Operation amplifier. The phase however does not coincidence with the theory as earlier discussed.

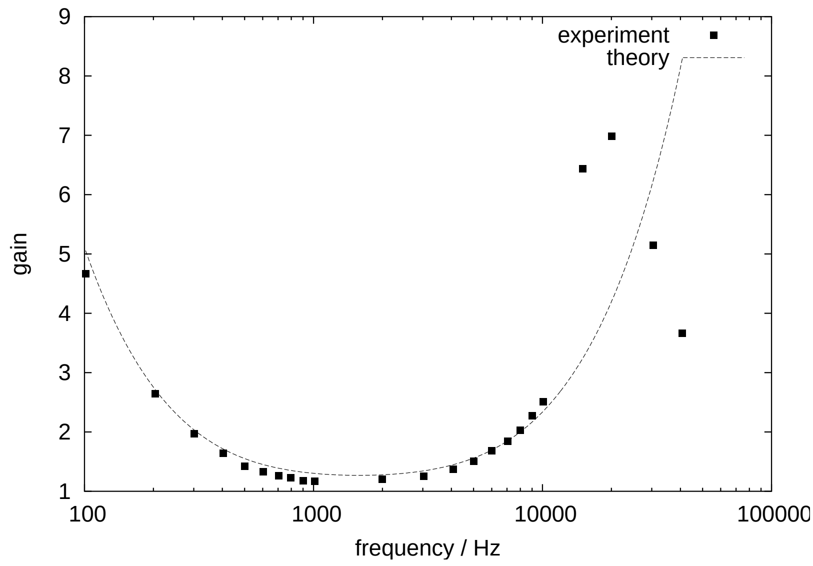


Figure 13: gain spectrum of PID

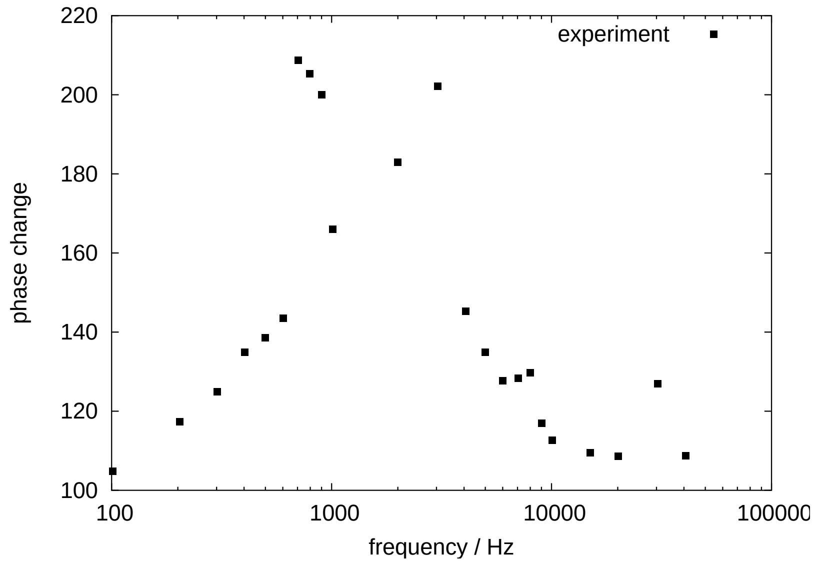


Figure 14: phase change spectrum of PID

3. Digitizing and spectral analysis

3.1. Fourier transformation (theory)

3.1.1. Fourier series

The general idea behind Fourier series is that, any periodic function $f(x+T_p) = f(x)$ can be expressed as an infinite series of harmonic components.

different notations

1. normal

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos \omega_n t + b_n \sin \omega_n t)$$

2. complex format

$$f(t) = \frac{a_0}{2} + \sum_{n=-\infty}^{\infty} c_n \mathrm{e}^{i\omega_n t} \qquad \text{with} \quad c_n = \frac{1}{T} \int\limits_{-T/2}^{T/2} f(t) \mathrm{e}^{-i\omega_n t} \, \mathrm{d}t$$

3. amplitude/phase format

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} c_n \sin(\omega_n t + \Psi_n)$$

amplitude: $c_n = \sqrt{a_n^2 + b_n^2}$
phases: $\tan \Psi_n = a_n / b_n$

Due to the orthogonal relations of the sine and cosine functions the coefficients can be expressed as

$$a_n = \frac{2}{T} \int\limits_{-T/2}^{T/2} f(t) \cos \omega_n t \, \mathrm{d}t$$

$$b_n = \frac{2}{T} \int\limits_{-T/2}^{T/2} f(t) \sin \omega_n t \, \mathrm{d}t$$

$$a_0 = \frac{2}{T} \int\limits_{-T/2}^{T/2} f(t) \, \mathrm{d}t$$

with $\omega_n = n \cdot \frac{2\pi}{T}$

3.1.2. Fourier transformation

For nonperiodic signals and for sections of periodic signals one uses the Fourier transformation instead of the Fourier series. The Fourier transformation and its backtransformation are defined as follows

$$H(\omega) = \frac{1}{\sqrt{2\pi}} \int\limits_{-\infty}^{\infty} \mathrm{e}^{-i\omega t} h(t)\,\mathrm{d}t$$

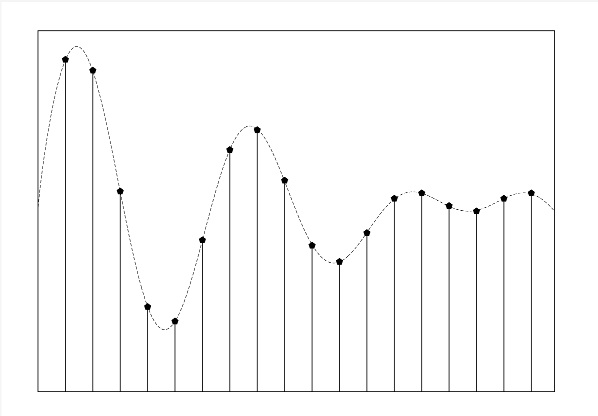
$$h(t) = \frac{1}{\sqrt{2\pi}} \int\limits_{-\infty}^{\infty} \mathrm{e}^{i\omega t} H(\omega)\,\mathrm{d}\omega$$

A very common way to describe the Fourier transformation is the following

$$\mathcal{F}\left\{h(t)\right\} = H(\omega)$$

3.1.3. discrete Fourier transformation

In the most common situtation, the signal (denoted with $h(t)$) is sampled (i.e., its value is recorded) at evenly spaced intervals in time. Let Δ denote the time interval between consecutive samples. The reciprocal of the time interval Δ is called the sampling rate.



The idea of discrete Fourier transformation is to estimate the Fourier transform of a function from a finite number of its sampled points. We can suppose that we have N consecutive sampled values h_k, at $k = 0, 1, 2, \ldots, N-1$ and denote the interval Δ.

With N numbers of input, we will evidently only be able to produce no more than N independent number of output. So, instead of trying to estimate the Fourier transform $H(\omega)$ at all values of ω, we seek estimates only at the discrete values:

$$\nu_n \equiv \frac{n}{N\Delta} \qquad n = -\frac{N}{2}, \ldots, \frac{N}{2}$$

The remaining step is to approximate the integral by a discrete sum:

$$\mathcal{F}\left\{h(t)\right\} = \int\limits_{-\infty}^{\infty} h(t)\mathrm{e}^{i\omega_n t}\,\mathrm{d}t \approx \sum_{k=0}^{N-1} h_k \mathrm{e}^{i\omega_n \Delta}\Delta = \Delta \sum_{k=0}^{N-1} h_k \mathrm{e}^{ikn/N}$$

The discrete Fourier transform maps N complex numbers (the h_k's) into N complex numbers (the H_k's) It does not depend on any dimensional parameter, such as the time scale Δ.

3.1.4. fast Fourier transformation

The fast Fourier transform (FFT) is a discrete Fourier transform algorithm which reduces the number of computations needed for N points from $2N^2$ to $2N \lg N$, where lg is the base-2 logarithm. The increase of speed relies on the avoidance of multiple calculations of values that cancel out each other. [1, 2]

3.2. Sampling rates / sampling theorem

The question is, what is the lowest sampling rate at which the signal can be reconstructed error-free? One would expect that if the signal has significant variation then the interval Δ must be small enough to provide an accurate approximation of the signal. Significant signal variation usually implies that high frequency components are present in the signal. It could therefore be inferred that the higher the frequency of the components present in the signal, the higher the sampling rate should be. [7]

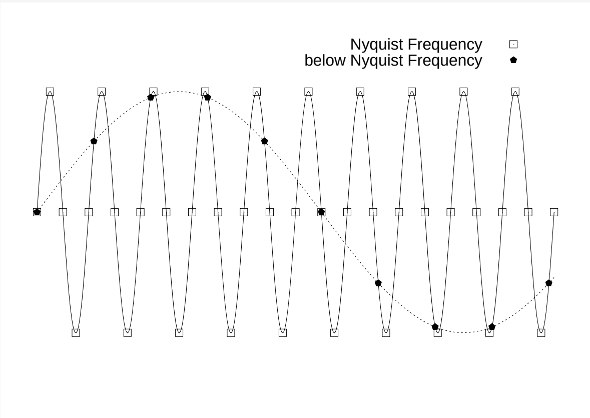
3.2.1. Nyquist Frequency

For any sampling interval Δ there is a critical frequency called the Nyquist Frequency given by

$$\nu_c \equiv \frac{1}{2\Delta} = 2\nu_{\text{signal}} \qquad (1)$$

which means, that it is necessary to sample more than twice as fast as the highest waveform frequency ν. This is the cutoff frequency above which a signal must be sampled in order to be able to fully reconstruct it. [4]

This also implies that no information is lost if a signal is sampled at the Nyquist frequency, and no additional information is gained by sampling faster than this rate.



3.2.2. Sampling theorem

Under Fourier transformation the sampling of a nonperiodic function will be mapped into a periodic function with periodicy of the sampling frequency ν_s. Thus the spectrum is identical with the original function in the range $-\frac{1}{2}\nu_s \leq \nu \leq \frac{1}{2}\nu_s$ as shown in figure 15. It follows thereby that the sampling frequency must be chosen so high, that the periodically recurring spectra do not overlap. This is called the 'sampling theorem'.

The sampling theorem states that a band-limited baseband signal must be sampled at a rate $\nu \geq 2B$. (B: Bandwidth) to be reconstructed fully. If the sampling rate is not high enough to sample the signal correctly then a phenomenon called aliasing occurs. [5]

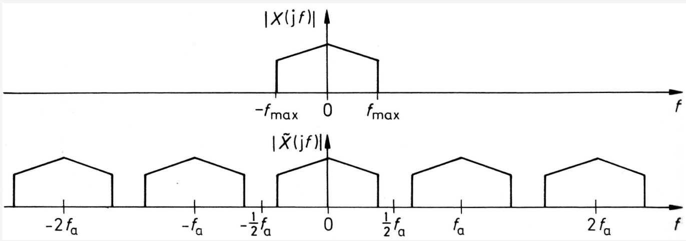


Figure 15: spectrum of input before and after sampling. ($f_a = \nu_S$) [8]

We record a sine shaped signal with the computer oscilloscope using different sampling rates, and take record of the frequency (resp. the time interval). The signal frequency used is approximately 1 kHz (1.0083 kHz).

Note: The sampling rate presented by the oscilloscope is in units of ms / division. Since it has 10 divisions the whole sampling range has been corrected by a number of ten.

sampling rate / ms	Δ T / ms	frequency
0.02	1.004	0.996 kHz
0.05	0.990	1.010 kHz
0.1	1.005	0.995 kHz
0.2	0.984	1.016 kHz
0.5	1.000	1.000 kHz
1	1.010	0.990 kHz
2	0.940	1.064 kHz
5	219	4.566 Hz
10	110	9.132 Hz
20	189	5.291 Hz
50	286	3.497 Hz

Table 2: frequency for different sampling rates

The Nyquist Frequency for 1 kHz is $\Delta = \frac{1}{2\nu} = 0.5\,\text{ms}$. In table 2 we can see that we measure correct frequencies for 10 times higher values. Below the sampling rate of 0.5 ms however the sine shape breaks down completely. Far below the Nyquist Frequency we get a sine shape signal again, but now with a 1000 times lower frequency. This is due to aliasing. This effect is shown on page 17.

Exemplarily we have taken pictures of the oscilloscope for sampling rates of 0.1, 0.5, 2, 100 ms. They are labelled with page 1 to 4 and can be found in the appendix.

3.3. Aliasing

Given a power spectrum (a plot of power vs. frequency), aliasing is a false translation of power falling in some frequency range $(-f_c, f_c)$ outside the range. Aliasing can be caused by discrete sampling below the Nyquist frequency. [3]

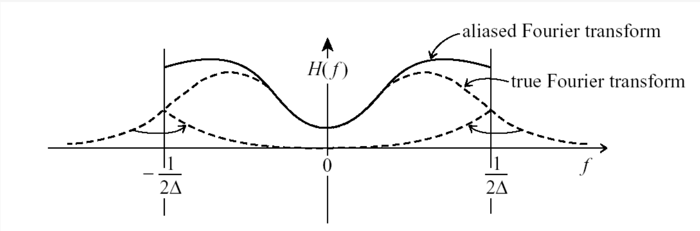


Figure 16: Example of aliased spectrum

We setup a bandwidth of 2 kHz and a frequency of 1 kHz and increased then the frequency slowly but continuously up to 5 kHz. Inbetween we have recorded some sample frequencies. They are shown in table 3.

frequency / kHz		
real	measured	distance
1.00	1.00	0.00
2.53	1.47	1.06
3.01	0.99	2.02
3.52	0.48	3.04
4.03	0.03	4.00
4.51	0.51	4.00
5.06	1.06	4.00

Table 3: Aliasing

What we see now, if one increases the frequency above the bandwidth is, that it seems that the peak of the spectrum bounces back from each side of the wall. This is due to the aliasing introduced above. Table 3 shows this behaviour very clearly. The distance between the real and the measured value is n-times half of the bandwidth which coincidences with saying that the peak is mapped back into the range if it exceeds the bandwidth limit.

Furthermore we have taken a look at the spectrum of a rectangular wave under conditions of aliasing. The frequency is 2 kHz. Apart from the center frequency ω_0 we get additional peaks with declining amplitude at $3\omega_0, 5\omega_0, 7\omega_0$ and so forth. Page 6 (in the appendix) shows the spectrum with a bandwidth of 2 kHz. Because of the backreflection all peaks appear in the center and form the high background. A similarly spectrum can be found with a bandwidth of 500 Hz (page 7). Here all the peaks overlap near zero because it is half of the original frequency.

The false spectrum becomes even more obvious when we use a bandwidth which is not two times an integer number of the frequency as with $\nu = 338.81$ Hz and badnwidth $B = 5$ kHz. This is shown on page 8. The in-between peaks have their origin in the aliasing effect and lead thus to an false spectrum. This behaviour is even increased with a bandwidth of 100 Hz and frequency of 1 kHz (page 9). Here the difference between the 'main' peaks amounts only 5 Hz instead of 2000 Hz !

At least we take a look at the spectrum of a triangular signal with frequency of 1 kHz and bandwidth 2 kHz (page 10). One should expect a picture like the one on page 6 for the rectangular signal. This picture however can easily be mistaken with sine signal as on page 5. The faster decrease of the amplitude leads to a much lower background, so that we see only one peak.

The shown examples demonstrate very clearly why it is very important to choose the correct bandwidth to record the correct spectrum. To get around of the aliasing effect filters are very common to reduce the bandwidth-passing frequencies.

3.4. Spectral analysis of sine, rectangular and triangular signals

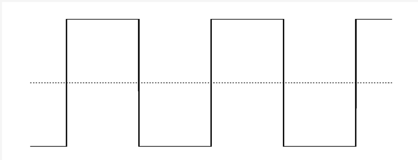
3.4.1. sine

A perfect sine signal with 1 kHz is sampled with a bandwidth of 2 kHz. The recorded spectrum can be seen on page 5 of our records. It shows one peak at the expected 1 kHz frequency. The observation is thus identical with the expected mathematical Fourier representation.

3.4.2. rectangular

rectangular sawtooth

$$f(t) = \begin{cases} -A & -\frac{T}{2} < t < 0 \\ A & 0 < t < \frac{T}{2} \end{cases}$$



The Fourier components are calculated using a computer-based algebra solution.

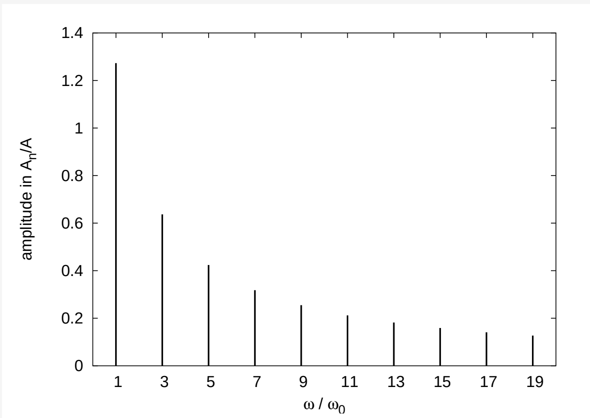
$$a_0 = 0$$
$$a_n = 0$$
$$b_n = \frac{2A}{\pi n}\big(\cos(n\pi) - 1\big) \qquad \Rightarrow \; n = 1, 3, 5, \ldots$$

The first factor vanishes because the function is symmetric around the y-axis. The second component is zero because the function is even.

Thus follows

$$f(t) = \frac{2A}{\pi} \sum_{n=1}^{\infty} \frac{1}{n} \sin\big((2n-1)\omega_0 t\big) \tag{2}$$

This means that we observe a spectrum with declining amplitude by $\frac{1}{n}$ in distances of $2\omega_0$.



We have taken records of a rectangular signal with 1 kHz on a range of 10 kHz. The peaks with amplitude and frequency are shown in table 4. A corresponding picture is on page 11 in the appendix. The accuracy of the data is 20 Hz and 16 mV.

ν /kHz	amplitude / V
0.996	3.484
3.008	1.250
5.020	0.625
7.031	0.578
9.004	0.406

Table 4: rectangular sawtooth spectrum

Figure 17 shows the corresponding plot. The $\frac{1}{n}$ decrease of the amplitude can be shown although it is not perfectly matched with the data. Likewise the spectrum is shown very well. We see peaks with distances of $2\omega_0$ at $\omega_0, 3\omega_0, 5\omega_0$ and so forth.

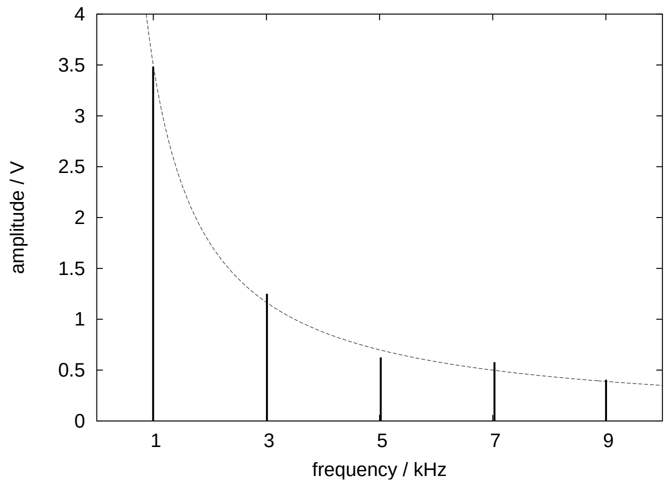
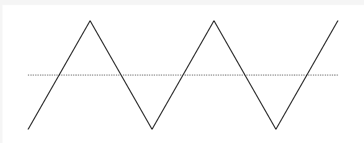


Figure 17: rectangular sawtooth spectrum

3.4.3. triangular

triangular sawtooth

$$f(t) = \begin{cases} \frac{4A}{T}t & 0 < t < \frac{T}{4} \\ 2A(1 - \frac{2}{T}t) & \frac{T}{4} < t < \frac{3}{2}T \\ -4A(1 - \frac{1}{T}t) & \frac{3}{4}T < t < T \end{cases}$$



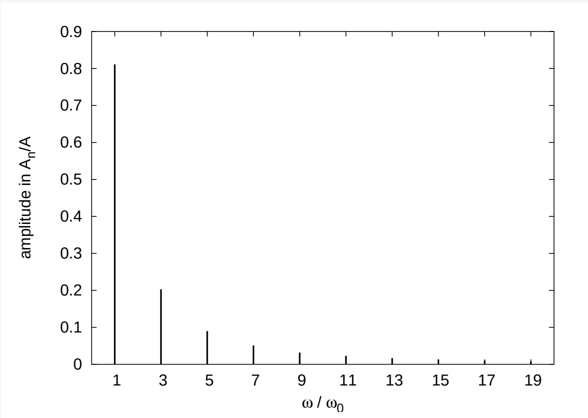
The Fourier components are calculated using a computer-based algebra solution.

$$a_0 = 0$$
$$a_n = 0$$
$$b_n = \frac{8A}{n^2\pi^2}\sin\left(1/2n\pi\right) \qquad \Rightarrow \; n = 1, 3, 5, \ldots$$

Thus follows

$$f(t) = \frac{8A}{\pi^2}\sum_{n=1}^{\infty}(-1)^{2n+1}\frac{1}{n^2}\sin\big((2n-1)\omega_0 t\big) \tag{3}$$

The spectrum differs from the rectangular sawtooth mainly by the faster decline of $\frac{1}{n^2}$.



We have taken records of a triangular signal with 1 kHz on a range of 10 kHz. The peaks with amplitude and frequency are shown in table 5. A corresponding picture is on page 12 in the appendix.

ν /kHz	amplitude / mV
1.016	2219
3.008	266
5.000	78.10
7.031	46.87
8.984	31.25

Table 5: triangular sawtooth spectrum

Figure 18 shows the corresponding plot. The $\frac{1}{n^2}$ decrease of the amplitude is perfectly matched. Likewise the spectrum is shown very well. We see peaks with distances of $2\omega_0$ at $\omega_0, 3\omega_0, 5\omega_0$ and so forth.

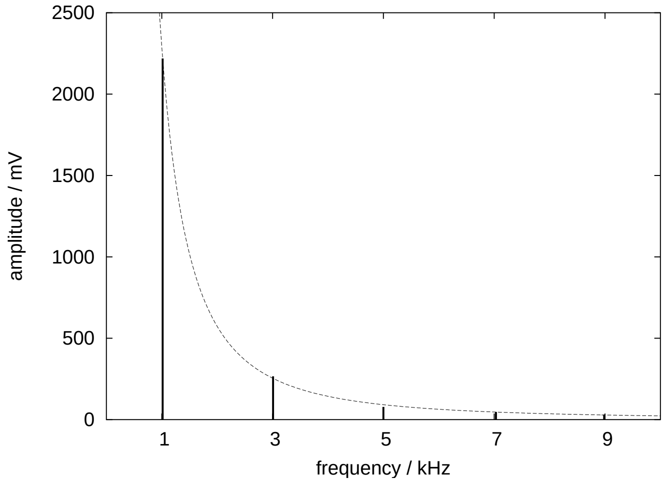


Figure 18: triangular sawtooth spectrum

3.5. Overloading of the Op-amplifiers

We setup the Inverting amplifier as it had been constructed in section 2.1 and connect the output to the oscilloscope/spectrum analyser. The input signal is a sine from the function generator. To overload the amplifier we simply increase the output amplitude of the function generator. What then happens is that the sine curve changes to a rectangular curve, because the upper and lower part is cut-off. Inbetween the shape is not a perfect rectangular . This can be observed in the spectrum analyser. When the signal starts to overload (not a perfect rectangular) the spectrum has peaks at $\omega, 2\omega, 3\omega$ and so forth, decreasing with frequency. Under increase of the output signal the shape becomes more and more a perfect rectangular and in the spectrum the even peaks decrease until they vanish completely whereas the odd peaks increase.

4. Modulation

4.1. Frequency modulation (FM)

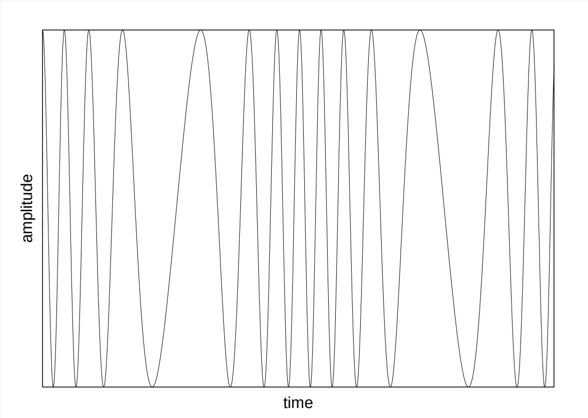


Figure 19: example of FM modulation with $\omega_c = 15$, $\omega_m = 2$, $\beta = 5$

A Frequency Modulated wave is a sine wave with a periodically varying instantaneous frequency and a constant amplitude. The average frequency is called the carrier frequency and the instantaneous frequency changes at the modulation frequency. The maximum excursion of the instantaneous frequency from the average is related to the modulation index.

Genrerally a signal is described as

$$S(t) = A\cos\big(\Phi(t)\big)$$

with amplitude A and phase $\Phi(t)$. The frequency is defined as

$$\omega \equiv \frac{\mathrm{d}\Phi}{\mathrm{d}t}$$

In case of frequency modulation this is

$$\omega = \omega_c + \Delta\omega m(t)$$

with carrier ω_c, modulation frequency $m(t)$ and its amplitude $\Delta\omega$. Accordingly is the signal is of the form

$$S(t) = A\cos\left(\omega_c t + \Delta\omega \int\limits_0^t m(t)\,\mathrm{d}t\right)$$

It is assumed, that the signal $m(t)$ is normalized so that the maximum of the integral is one.

For a modulation signal of the form

$$m(t) = \cos(\omega_m t)$$

the time dependent frequency is

$$\omega = \omega_c + \Delta\omega \cos(\omega_m t)$$

and the phase

$$\Phi = \omega_c t + \frac{\Delta\omega}{\omega_m} \cos(\omega_m t)$$

The ratio $\beta = \frac{\Delta\omega}{\omega_m}$ is called the modulation index.

The entire expression is thus

$$S(t) = A\cos\big(\omega_c t + \beta \sin(\omega_m t)\big) \tag{4}$$

The frequency spectrum can be found by rewriting the above expression as a sum of components of constant frequency using the properties of the Bessel Functions. This gives:

$$\begin{aligned} S(t) = A\big\{&J_0(\beta)\sin(\omega_c t) \\ &+ J_1(\beta)\,[\sin(\omega_c+\omega_m)t - \sin(\omega_c-\omega_m)t] \\ &+ J_2(\beta)\,[\sin(\omega_c+2\omega_m)t + \sin(\omega_c-2\omega_m)t] \\ &+ J_3(\beta)\,[\sin(\omega_c+3\omega_m)t - \sin(\omega_c-3\omega_m)t] \\ &+ \dots \end{aligned} \tag{5}$$

This expression implies that the FM spectrum consists of a component at ω_c and an infinite number of lines at $\omega_c \pm n\omega_m$ and that the amplitude of the components are given by the Bessel functions. [6]

We use both function generators to generate a frequency modulated signal. Thereby one generator is the input for the other generator.

To observe the effect of frequency modulation on the oscilloscope we scanned a few varieties of frequencies and modulation indices. Finally we have set up a carrier frequency of 25 kHz and a modulation frequency of 625 Hz with a high amplitude of the modulation signal (high modulation index). The plot can be found on page 13 in the appendix. In the range where the frequency goes to zero the modulation value is the highest, whereas in the other range the wave of the modulation goes through zero and thus the frequency is more or less the carrier frequency. If one increases the amplitude the number of periods will increase, but the overall shape stays the same. The according spectrum has equal high and low frequencies (white spectrum).

To observe a typical FM spectrum we had to reduce the amplitude of the modulation signal by 20 dB. The carrier frequency is now set to 25 kHz and the modulation frequency to 5 kHz. If we vary the modulation index (resp. the modulation-amplitude) we find that with low modulation index we have only the peak from the carrier signal. Whereas when we increase the modulation index the carrier peak in the middle decreases and symmetrically peaks appear on both sides growing with frequency. This behaviour is shown on pages 14-16 in the appendix. (Note: The printout of the last page is did not match with the screen. The low amplitude of the peaks is due to the program or the printer)

Finally we are interested in the spacing between the peaks. The expansion into Besselfunctions shows that we should expect an equal spacing between the peaks by the modulation frequency independently of the modulation index. Therefore we have measured the spacing for different indices. The data is shown in table 6. One can see clearly that the spacing is according to the theory. Pages 14-16 show the according spectrum.

	frequency peak						
modulation index	ω_1	ω_2	ω_3	ω_4	ω_5	ω_6	ω_7
$8.000 \cdot 10^{-5}$		15.82	20.8	25.78	30.76	35.84	
$1.876 \cdot 10^{-4}$	10.44	15.43	20.5	25.58	30.56	35.64	40.62
$2.218 \cdot 10^{-4}$	10.64	15.72	20.7	25.87	30.76	35.84	40.91
	frequency peak spacing						
	$\omega_2 - \omega_1$	$\omega_3 - \omega_2$	$\omega_4 - \omega_3$	$\omega_5 - \omega_4$	$\omega_6 - \omega_5$	$\omega_7 - \omega_6$	
$8.000 \cdot 10^{-5}$		4.98	4.98	4.98	5.08		
$1.876 \cdot 10^{-4}$	4.99	5.07	5.08	4.98	5.08	4.98	
$2.218 \cdot 10^{-4}$	5.08	4.98	5.17	4.89	5.08	5.07	

Table 6: modulation index

4.2. Amplitude modulation (AM)

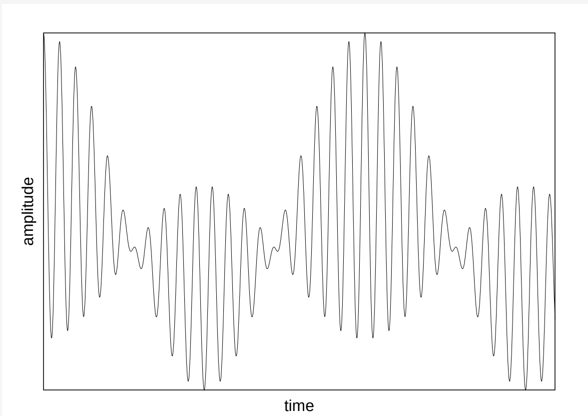


Figure 20: Example of AM modulation with offsets in carrier and signal

In Amplitude Modulation or AM, the carrier signal

$$S_c(t) = A_0 + A\cos(\omega_c t)$$

has its amplitude A modulated by the (lower frequency) message signal $m(t)$. We assume a harmonic oscillating modulation

$$m(t) = B_0 + B\cos(\omega_m t)$$

Then follows for the linear amplitude modulated signal

$$\begin{aligned} S(t) &= m(t) \cdot S(t) \\ &= A_0 B\cos(\omega_m t) + B_0 A\cos(\omega_c t) + AB\cos(\omega_c t)\cdot\cos(\omega_m t) \\ &= A_0 B\cos(\omega_m t) + B_0 A\cos(\omega_c t) \\ &\quad + \frac{1}{2}AB\cos\big((\omega_c+\omega_m)t\big) + \frac{1}{2}AB\cos\big((\omega_c-\omega_m)t\big) \end{aligned} \tag{6}$$

Hence we observe the carrier and modulation signal if their offsets are not zero and additionally we see signals at frequencies $\omega_c + \omega_m$ and $\omega_c - \omega_m$.

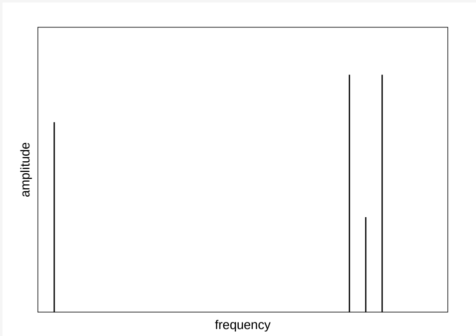
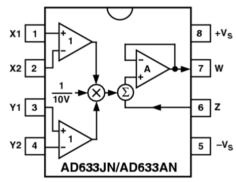


Figure 21: spectrum of AM modulation with offsets in carrier and signal



In the following experiments we are working with a low cost analog multiplier ANALOG DEVICES AD633. This allows to add and multiply signals. Hence we can use it to do the amplitude modulation.

In the beginning we had a serious problem with the AD633 device. In the end it was not clear what the reason was, since we had disconnected and checked everything and then connected everything in the same way. To achieve amplitude modulation one uses the noninverting inputs 1 and 3 on the device.

On page 17 we have printed a demonstration of amplitude modulation with a carrier frequency of 15 kHz and a modulation of 1 kHz.

To verify equation 6 for the amplitude modulation we have set the offsets A_0 and B_0 to non-zero values and changed the amplitude B_0 of the modulation frequency. Thus all peaks should increase except for the carrier frequency peak. Table 7 and the according figure 22 prove this behaviour.

modulation amplitude / V	amplitude / mV			
	ω_m	$\omega_c - \omega_m$	ω_c	$\omega_c + \omega_m$
5.00	207	437	235	465
5.94	274	549	240	588
6.94	319	667	240	712
8.13	369	774	240	819

Table 7: dependence on modulation amplitude

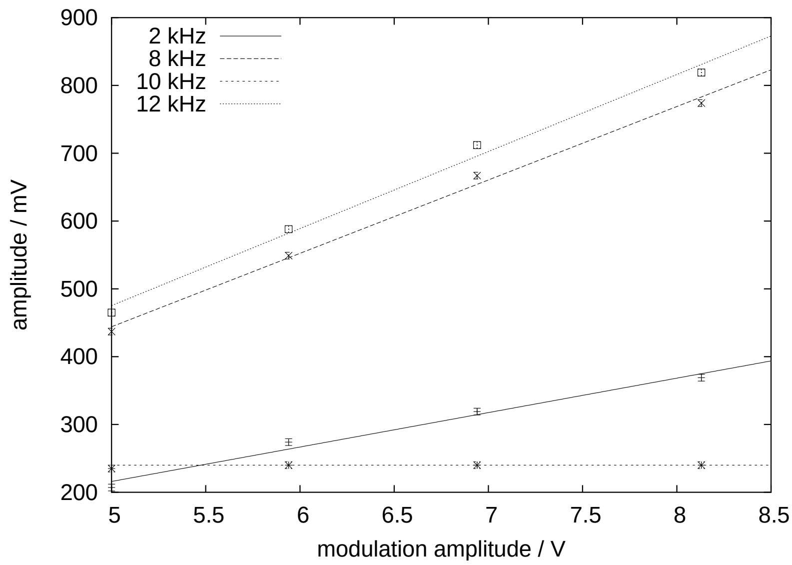


Figure 22: dependence on modulation amplitude

Finally we wanted to see that the peaks of the carrier and modulation frequencies decrease to zero without offsets on the signals. This is shown on pages 18-20 in the appendix. First with offset on both signals, second with no offset on the carrier (modulation peak vanishes) and third with no offset and hence no carrier or modulation peak.

4.3. Comparison of both modulation techniques

Frequency modulation (FM) depends strongly on the modulation index whereas the amplitude modulation (AM) only depends on the amplitude. In FM the spectrum consists of many peaks around the carrier where the number depends on the index. Their amplitudes is determined by besselfunctions and thus has no linear dependence on the input amplitude. The spacing between the peaks is determined by the modulation frequency.

Amplitude modulation contrary has two peaks in the spectrum around the carrier and if they have offsets a peak at the carrier and modulation frequency itself. That means especially that it is possible to have a spectrum with no frequency of the input signals! The amplitude depends linearly on the amplitude of the input signals.

5. Noise

Loosely, noise is a disturbance tending to interfere with the normal operation of a device or system. It is an undesired disturbance within the frequency band of interest, that affects a signal and may distort the information carried by the signal.

5.1. Different noise processes

A noise is a random signal of known statistical properties of amplitude, distribution, and spectral density. Generally speaking, there are five different kinds of noise: *thermal noise, shot noise, $1/f$ noise (technical noise), generation and combination noise especially in semiconductors, and white noise*
Although the production mechanisms vary, generally, the noises are produced in a certain period of time t, called the characteristic time. And the sampling time is T, f is the sampling frequency, defined by $f = 1/T$. So for different t and T relationships, the spectral dependence of the noise power $\omega(f)$ is different.

1. $t \ll T : \omega(f) \propto f^0$
2. $t \gg T : \omega(f) \propto f^{-2}$
3. $t \approx T : \omega(f) \propto f^{-1}$

For example, 1 can be white noise, 2 can be generation and recombination noise, 3 can be technical noise $P(f) \propto f^{-1}$, for the 3, since all the noise processes should have almost the same characteristic time, so the spectral range is quite narrow.

Thermal noise

The noise generated by thermal agitation of electrons in a conductor. The noise power is given by

$$W = kT\Delta f$$

W noise power in watt, k Boltzmann's constant in joules per kelvin, T conductor temperature in kelvin, Δf bandwidth in hertz. Especially for a RC filter

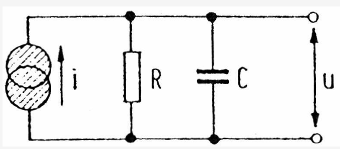


Figure 23: circuit of RC filter

$$W = kT\frac{4R}{1 + (fRC)^2}$$

This spectrum is shown in figure 24.

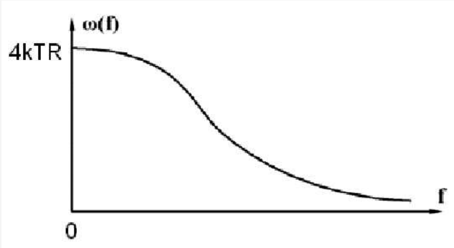


Figure 24: spectrum of thermal noise in RC filter circuit

Shot noise

Shot noise is the time dependent fluctuations in the electrical current due to the discreteness of the electron charge. For example, in high vacuum diode, the applied voltage is so high that electrons are injected to anode A in a quite short period of time, the current reaches maximum; then the high vacuum diode is saturated; after a relax time, the next ,impulse'. So at the frequency $f = 1/t$, the amplitude is 0, as can be seen in figure 26.

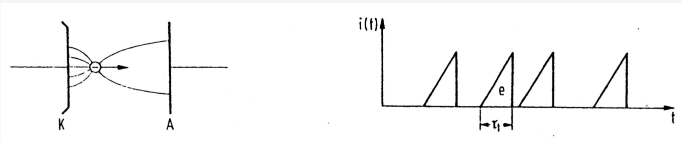


Figure 25: Mechanism of shot noise

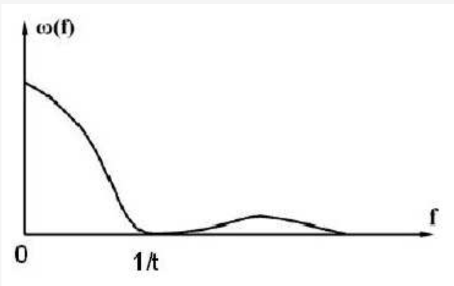


Figure 26: spectrum of shot noise

$1/f$ noise(technical noise)

The power of this kind of noise decrease proportional to 1/f, as can be seen in figure 27. 1/f noise appears in nature all over the places. It's difficult to deal with it, if you perform a measurement at low frequencies.

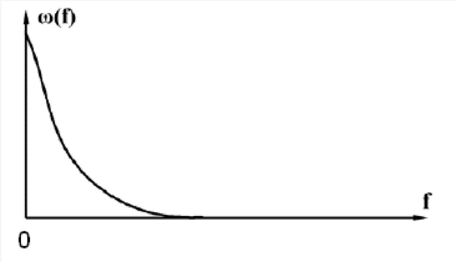


Figure 27: spectrum of technical noise noise in RC filter circuit

Generation and recombination noise

In semiconductors, there is a generation-recombination process. In the semiconductors, carriers are freed from association with a particular atom by a generation process, which induces the conduction. The ‚uncovered' atoms will trap carriers. Because of the thermal energy of the crystal lattice, the trapped carrier will be freed again after only a short time. This process is a series of independent discrete events. Each event causes fluctuation in the number of free carriers leading to a fluctuation in the material resistance.

The frequency response is constant at low frequency with a corner at a frequency $f = 1/2\pi t$. Above this corner the high frequency slope is proportional to $1/f^2$, as can be seen from figure 28.

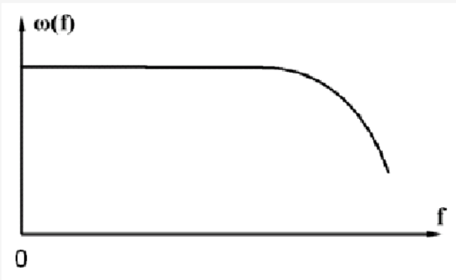


Figure 28: spectrum of generation and recombination noise

White noise

The noise power does not depend on frequency over a very wide frequency range, see for example figure 29.

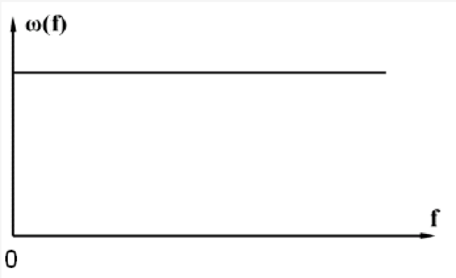


Figure 29: spectrum of white noise

5.2. Spectral properties of the noise generator

We connect the output of the noise generator to the input of the spectrum analyser. In the frequency spectrum, it can be seen that the amplitude of the power P is proportional to $\frac{1}{f}$. It is therefor technical noise, as can be seen from printed graph 21.

5.3. Methods to improve the signal to noise ratio

We generate a signal of frequency 5.6kHz, and then generate the noise, with a signal to noise ratio about 1. There are two ways to improve the signal to noise ratio.

First, average over the spectrum, since the noise obeys a ramdon distribution, the mean value tends to 0 as more spectra averaged; but the intensity of signal at a certain frequency remains the same.

Second, a band pass filter is used to cut the noise, and then average the output from the filter. We use the fast Fourier transformation spectrum, and average tool. The effect of adding a filter is quite obvious, as can be seen from figure 32.

calculation of bandpass filter

The circuit used for the bandpass filter is shown in figure 30.

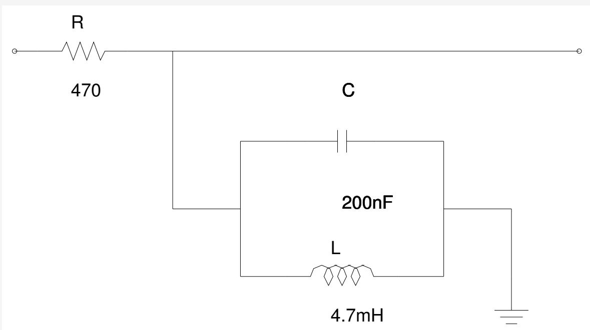


Figure 30: Bandpass filter circuit

The current in the input and output circuit must be the same. This yields to

$$\frac{U_a}{U_i} = \frac{Z_{LC}}{R + Z_{LC}} = \frac{1}{1 + R/Z_{LC}} = \frac{1}{1 + R\left(i\omega C + \frac{1}{i\omega L}\right)} = \frac{1}{1 + iR\left(\omega C - \frac{1}{\omega L}\right)}$$

The absolute value of this is

$$\left|\frac{U_a}{U_i}\right| = \frac{1}{1 + R^2\left(\omega C - \frac{1}{\omega L}\right)^2} = \frac{1}{1 + \omega^2 R^2 C^2 \left(1 - \frac{\omega_0^2}{\omega^2}\right)^2}$$

with resonance frequency $\omega_0 = \frac{1}{\sqrt{LC}}$. The bandwidth is determinded by $\Delta\omega = \frac{1}{RC}$. The characteristic of this filter is shown in figure 31

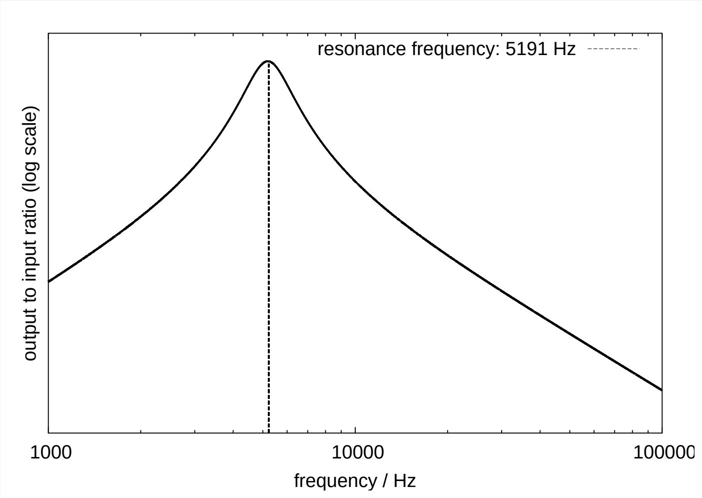


Figure 31: Bandpass characteristic

The electronic devices we have used are:

1. $R = 470\,\Omega$
2. $C = 200\,\mathrm{nF}$
3. $L = 4.7\,\mathrm{mH}$

This leads to a center frequency of $\omega_0 = 5191\,Hz$

How to measure the intensity of noise and signal?

Here just the region on both sides near the signal peak is measured, we get a high value where most positive noise peaks are and a low value for the negative noise vallies, and substrate the two values to get the difference. For each kind of average, four groups of data are recorded and the mean value is calculated as the intensity of noise. And the middle of noise intensity is used as the background for the correspond signal's peak, the difference between the signal peak and the 0 point is the intensity of the peak. Also four groups of data are recorded to calculate the mean value. The result can be seen in table 8.

It can be seen clearly that the average method can improve the signal to noise ratio, but the filter improve the ratio much more significantly, so that the latter method should be used whenever possible. For the filter method, after average over 20 times, the slope is smaller than before, since more spectra are used to average, the longer time it takes, so we think using the filter just average over 20 times is enough.

	signal to noise ratio	
average times	average	average+filter
1	0.97	12.60
2	1.67	16.39
5	3.11	29.28
10	4.27	53.13
20	5.75	69.63
50	9.74	104.50

Table 8: Improve signal to noise ratio

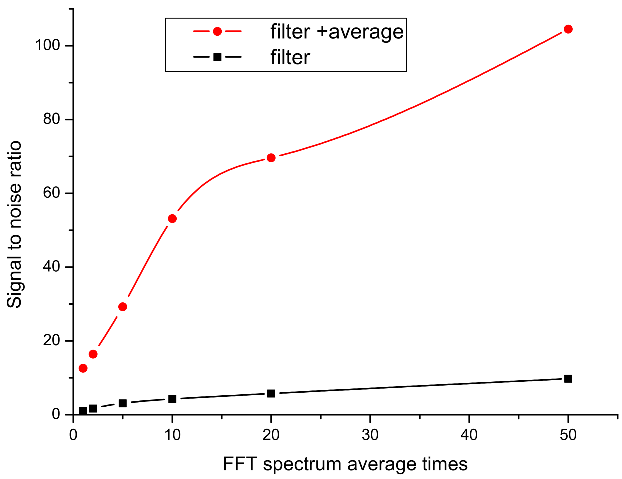


Figure 32: Signal to noise ratio

5.4. Correlation of noise

Here we have two noise generators 1 and 2, and a function generator. The correlation is viewed in the oscilloscope on the x-y plot, with one cable connected to the x and the other to the y.

Firstly, we use noise generators 1 and 2 to have two independent noise signals, connect the output signals with cables channel 1 and channel 2. The graph is irregular, the two has no correlation. As can be seen from printed graph 23.

Secondly, we use noise generator 1's output as noise generator 2' input, and connect the output signals of 1 and 2 with oscilloscope cables. We find the fluctuation is in a certain direction, they are part correlated. As can be seen from printed graph 24.

Thirdly, we use the function generator's output as noise generator 1's input, and connect the both outputs of function generator and noise to the cables. When the amplitude of the noise is smallest, there is a line in the screen, the slope of line changes as the signal's frequency changes. This means that the two output totally correlate, and the phase shift changes with the function's frequency. The amplitude of the noise here is so small that it can not disturb the signal of function generator correlating with itself, its only effect is a constant phase shift. When increase the amplitude of the noise, the shape gradually changes from line to ellipse, now they are partially correlated. When we continuously increase the noise amplitude, the graph is irregular again, they are not correlated at all. As can be seen from printed graph 25.

A. measuring data

A.1. Operational Amplifiers

frequency / kHz	gain	$\Delta T/\mu\text{s}$	phase shift / degree
0.01	10.00	$48.00 \cdot 10^3$	172.80
0.10	10.00	$4.92 \cdot 10^3$	177.12
1.00	10.00	490.00	176.40
10.00	10.00	47.20	169.92
20.00	9.063	31.00	223.20
40.50	4.563	16.80	244.94
60.20	3.063	11.40	247.06
80.30	2.406	6.30	182.12
100.10	2.000	2.72	98.02
150.20	1.359	1.75	94.63
200.90	1.016	1.20	86.79

Table 9: Inverting Operational Amplifier

frequency / kHz	gain	$\Delta T/\text{ms}$	phase shift / degree
10.55	43.34	25.6	97.23
20.46	24.67	13.4	98.70
30.75	17.00	9.2	101.84
40.58	12.67	6.7	97.88
50.86	10.17	5.4	98.87
60.90	8.83	4.4	96.47
70.00	7.67	3.9	98.78
80.49	6.40	3.4	98.52
90.84	5.66	3.0	98.11
100.32	5.21	2.7	98.23
200.80	2.67	1.3	91.08
299.20	1.79	0.8	87.25
400.40	1.33	0.6	87.93
505.20	1.03	0.5	87.3
601.80	0.83	0.4	86.66
703.10	0.73	0.3	84.03
798.60	0.62	0.3	83.95
1014.00	0.48	0.2	81.77

Table 10: Integrator circuit

frequency / kHz	gain	$\Delta T/\mathrm{ms}$	phase shift / degree
0.10	0.023	2.60	93.60
0.50	0.100	0.52	93.60
1.01	0.260	260.00	94.54
2.01	0.425	124.00	89.73
2.53	0.538	104.00	94.72
3.01	0.638	85.00	92.11
3.51	0.750	72.00	90.98
4.02	0.875	65.00	94.07
4.52	0.984	58.00	94.38
5.07	1.094	52.00	94.91
5.50	1.203	46.50	92.07
6.01	1.281	42.50	91.95
10.08	2.154	26.50	96.16
15.03	3.015	17.50	94.69
20.08	5.106	13.00	93.97
25.20	5.906	12.30	111.59
30.34	4.860	9.60	104.86
35.24	4.183	7.80	98.95
40.11	3.691	6.60	95.30
50.08	2.892	4.90	88.34
60.88	2.430	4.20	92.05
70.36	2.154	3.95	100.05
80.24	1.907	3.05	88.10
90.87	1.661	2.70	88.33
100.48	1.507	2.40	86.81
150.02	1.031	1.55	83.71
200.43	0.769	1.20	86.59

Table 11: Differentiator circuit

frequency / Hz	gain	$\Delta T/\mathrm{ms}$	phase shift / degree
101.1	4.67	2.880	104.82
203.7	2.64	1.600	117.33
301.8	1.97	1.150	124.95
402.9	1.64	0.930	134.89
500.0	1.42	0.770	138.60
604.1	1.33	0.660	143.53
703.8	1.26	0.824	208.78
796.5	1.23	0.716	205.31
902.1	1.18	0.616	200.05
1011	1.17	0.456	165.97
2001	1.20	0.254	182.97
3036	1.25	0.185	202.20
4077	1.37	0.099	145.30
4997	1.51	0.075	134.92
6011	1.68	0.059	127.67
7060	1.84	0.051	128.35
8007	2.03	0.045	129.71
9021	2.27	0.036	116.91
10095	2.51	0.031	112.66
15020	6.44	0.020	109.50
20110	6.98	0.015	108.59
30390	5.15	0.012	126.91
40795	3.66	0.007	108.68

Table 12: PID circuit

B. printout

page	description
1.	sampling rate: 0.1 ms
2.	sampling rate: 0.5 ms
3.	sampling rate: 2.0 ms
4.	sampling rate: 0.1 s
5.	sine signal
6.	rectangular signal; frequency 1 kHz; bandwidth 2 kHz
7.	rectangular signal; frequency 1 kHz; bandwidth 0.5 kHz
8.	rectangular signal; frequency 338.81 Hz; bandwidth 5 kHz
9.	rectangular signal; frequency 1 kHz; bandwidth 0.1 kHz
10.	triangular signal; frequency 1 kHz; bandwidth 2 kHz
11.	rectangular signal; frequency 1 kHz; bandwidth 10 kHz
12.	triangular signal; frequency 1 kHz; bandwidth 10 kHz
13.	frequency modulation
14.	frequency modulation spectrum with 5 kHz range; modulation index: 0.4 V
15.	frequency modulation spectrum with 5 kHz range; modulation index: 0.938 V
16.	frequency modulation spectrum with 5 kHz range; modulation index: 1.109 V
17.	amplitude modulation with 15 kHz and 1 kHz
18.	amplitude modulation spectrum with offset on both signals
19.	amplitude modulation spectrum with no offset on carrier signal
20.	amplitude modulation spectrum with no offset on both signals
21.	noise spectrum
22.	signal with noise spectrum with bandwidth filter
23.	correlation x-y plot: uncorrelated
24.	correlation x-y plot: partially correlated
25.	correlation x-y plot: correlated with 180 degree phase shift

Table 13: List of printed pages

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References

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